Packet #6

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Reteaching

8.6 Radical Expressions and Radical Functions

◆Skill A Evaluating cube-root expressions

Recall $x^{\frac{1}{3}} = \sqrt[3]{x}$ and $x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

◆ Example

Evaluate each expression.

a. $(\sqrt[3]{64})^2 + 7$ **b.** $5\left(\sqrt[3]{\frac{-108}{4}}\right)$ **c.** $\frac{1}{3}\sqrt[3]{27^2}$

◆ Solution

a. $(\sqrt[3]{64})^2 + 7 = 4^2 + 7$
$= 23$

b. $5\left(\sqrt[3]{\frac{-108}{4}}\right) = 5\sqrt[3]{-27}$
$= 5(-3)$
$= -15$

c. $\frac{1}{3}\sqrt[3]{27^2} = \frac{1}{3} \cdot 3^2$
$= 3$

Evaluate each expression without using a calculator.

1. $(\sqrt[3]{125^2})$ ____________
2. $\sqrt[3]{125^2}$ ____________
3. $(-8)^{\frac{1}{3}} + 2$ ____________
4. $\frac{1}{3}\left(\sqrt[3]{\frac{81}{3}}\right)$ ____________
5. $\left[\frac{1}{2}\sqrt[3]{-216}\right]^{-1}$ ____________
6. $(27^4)^{\frac{1}{3}}$ ____________

◆Skill B Using transformations to graph square root functions

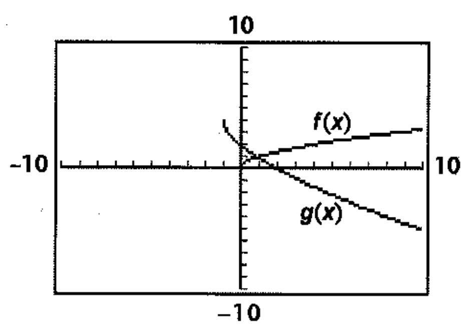
Recall The domain of a square root function is the set of all x such that the radicand (expression under the radical sign) is greater than or equal to 0.

◆ Example

For the function $g(x) = -3\sqrt{x + 1} + 5$

a. state the domain and

b. use transformations to sketch a graph.



◆ Solution

a. Since the radicand must be nonnegative, $x + 1 \geq 0$. The domain is $x \geq -1$.

b. Write $g(x) = -3\sqrt{x + 1} + 5$ in the form $g(x) = a\sqrt{b(x - h)} + k$, where
$|a|$ is the vertical stretch/compression factor,
$|b|$ is the horizontal stretch/compression factor,
h gives the horizontal translation, and
k gives the vertical translation.
In this function, $a = -3$, $b = 1$, $h = -1$, and $k = 5$.
$g(x) = -3\sqrt{1(x - (-1))} + 5$
Start with $\sqrt{x}$ and stretch vertically by a factor of 3; reflect across the x-axis (since $a < 0$); there is not a horizontal stretch; translate 1 unit to the left and 5 units up.

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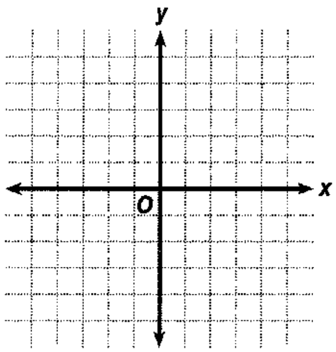
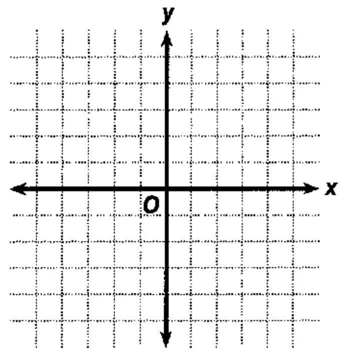
Find the domain and sketch a graph for each radical function. Check with a graphics calculator.

7. $f(x) = 2\sqrt{3(x-1)} - 3$ ______________



8. $f(x) = -2\sqrt{x+5} + 2$ ______________

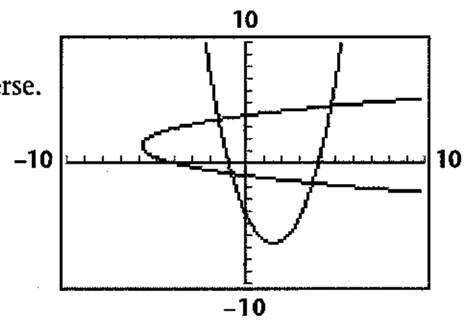


◆Skill C Finding and graphing the inverse of a quadratic function

Recall To find the inverse of a function, interchange x and y.

◆ **Example**

Find the inverse of $f(x) = x^2 - 3x - 4$.
Then graph both the function and its inverse.



◆ **Solution**

$x = y^2 - 3y - 4$

$y^2 - 3y - 4 - x = 0$

Let $a = 1$, $b = -3$, and $c = -4 - x$.

Use the quadratic formula to solve for y.

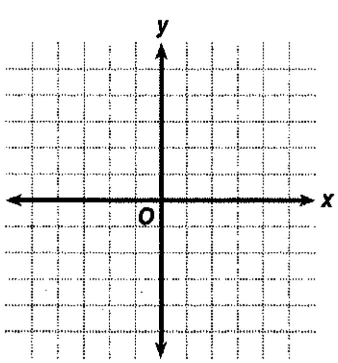
$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4-x)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{25 + 4x}}{2}$$

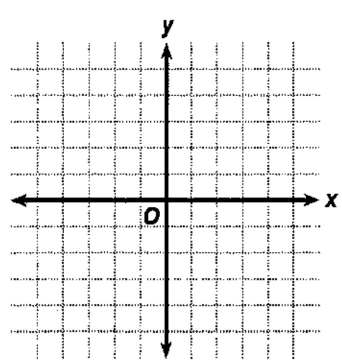
Graph $y_1 = x^2 - 3x - 4$, $y_2 = \frac{3 + \sqrt{4x + 25}}{2}$, and $y_3 = \frac{3 - \sqrt{4x + 25}}{2}$.

Find the inverse of each quadratic function. Then graph the function and its inverse in the same coordinate plane.

9. $f(x) = x^2 - 4$ ______________



10. $f(x) = x^2 + 3x$ ______________



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Reteaching
8.7 Simplifying Radical Expressions

◆Skill A Simplifying radicals

Recall If n is an even integer, then $\sqrt[n]{a^n} = |a|$; for example, $\sqrt[4]{(-2)^4} = |-2| = 2$.
If n is an odd integer, then $\sqrt[n]{a^n} = a$; for example, $\sqrt[3]{(-2)^3} = -2$.

◆ Example

Simplify $\sqrt{50a^3bc^4}$.

◆ Solution

$\sqrt{50a^3bc^4} = \sqrt{25a^2c^4 \cdot 2ab}$ *Make one monomial a perfect square.*

$= \sqrt{25a^2c^4} \cdot \sqrt{2ab}$ *Product Property of Radicals*

$= 5|a|c^2\sqrt{2ab}$ *c^2 is always nonnegative. Thus, absolute value is not needed.*

Simplify each radical expression by using the Properties of *n*th Roots.

1. $\sqrt{48x^3y^4}$ ________ **2.** $\sqrt{8x^6y^5}$ ________ **3.** $\sqrt{150m^{12}}$ ________

4. $\sqrt{175a^2b^3c^4}$ ________ **5.** $\sqrt[3]{54x^6}$ ________ **6.** $\sqrt[5]{32r^{12}x^{10}}$ ________

◆Skill B Simplifying products and quotients of radical expressions

Recall Product Property of Radicals: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ where $b \neq 0$

◆ Example

Simplify each expression. **Assume** that the value of each variable is positive.

a. $(18xy^2)^{\frac{1}{2}} \cdot \sqrt{2xz^3}$ **b.** $\dfrac{(24x^3y^7)^{\frac{1}{3}}}{\sqrt[3]{3y^2}}$

◆ Solution

a. $(18xy^2)^{\frac{1}{2}} \cdot \sqrt{2xz^3} = \sqrt{18xy^2 \cdot 2xz^3}$ *Product Property of Radicals*

$= \sqrt{36x^2y^2z^2 \cdot z}$

$= \sqrt{36x^2y^2z^2} \cdot \sqrt{z}$

$= 6xyz\sqrt{z}$

No absolute value is necessary, since values of variables are assumed to be positive.

b. $\dfrac{(24x^3y^7)^{\frac{1}{3}}}{\sqrt[3]{3y^2}} = \sqrt[3]{\dfrac{24x^3y^7}{3y^2}}$

$= \sqrt[3]{8x^3y^5} = \sqrt[3]{8x^3y^3 \cdot y^2} = \sqrt[3]{8x^3y^3} \cdot \sqrt[3]{y^2} = 2xy\sqrt[3]{y^2}$

Simplify each product or quotient. Assume that the value of each variable is positive.

7. $\sqrt{6x^2} \cdot \sqrt{6x^2}$ ________ **8.** $\sqrt{3x^4} \cdot \sqrt{12x^3}$ ________ **9.** $(15x^4y^2)^{\frac{1}{2}} \cdot \sqrt{5y^6}$ ________

10. $\dfrac{\sqrt{54x^4y^6}}{\sqrt{6xy^4}}$ ________ **11.** $\dfrac{(216a^9)^{\frac{1}{3}}}{\sqrt[3]{a^6}}$ ________ **12.** $\dfrac{\sqrt[3]{108a^{15}b^{10}}}{(2b)^{\frac{1}{3}}}$ ________

◆ Skill C Performing operations on radical expressions

Recall By using "FOIL," $(a + \sqrt{b})(c + \sqrt{d}) = ac + a\sqrt{d} + c\sqrt{b} + \sqrt{bd}$.

◆ Example 1

Simplify by performing the indicated operations: $(2 + \sqrt{8})(5 - \sqrt{2}) + 3\sqrt{2}$.

◆ Solution

$$\begin{aligned}(2 + \sqrt{8})(5 - \sqrt{2}) + 3\sqrt{2} &= 10 - 2\sqrt{2} + 5\sqrt{8} - \sqrt{16} + 3\sqrt{2}\\ &= 10 - 2\sqrt{2} + 10\sqrt{2} - 4 + 3\sqrt{2}\\ &= 10 - 4 + \sqrt{2}(-2 + 10 + 3)\\ &= 6 + 11\sqrt{2}\end{aligned}$$

◆ Example 2

Rationalize the denominator of each expression. **a.** $\dfrac{8}{\sqrt{2}}$ **b.** $\dfrac{6}{\sqrt{3} + 2}$

◆ Solution

a. $\dfrac{8}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$ *Multiply by* $\dfrac{\sqrt{2}}{\sqrt{2}}$.

$= \dfrac{8\sqrt{2}}{2}$

$= 4\sqrt{2}$

b. $\dfrac{6}{\sqrt{3} + 2} = \dfrac{6}{\sqrt{3} + 2} \cdot \dfrac{\sqrt{3} - 2}{\sqrt{3} - 2}$

$= \dfrac{6\sqrt{3} - 12}{(\sqrt{3})^2 - (2)^2}$

$= \dfrac{6\sqrt{3} - 12}{-1}$

Find each sum, difference, or product. Give your answer in simplest radical form. Assume that the value of each variable is positive.

13. $(\sqrt{11} - \sqrt{13})(\sqrt{11} + \sqrt{13})$ ________ **14.** $(2\sqrt{x} + 1)(\sqrt{x} - 4)$ ________

15. $3\sqrt{7}(\sqrt{7} - \sqrt{14})$ ________ **16.** $(3\sqrt{y} - 1)(2\sqrt{y} + 4) - 10\sqrt{y}$ ________

Write each expression with a rational denominator and in simplest form.

17. $\dfrac{3}{\sqrt{2}}$ ________ **18.** $\dfrac{3\sqrt{7}}{7\sqrt{3}}$ ________ **19.** $\dfrac{4}{3 - 2\sqrt{2}}$ ________

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Reteaching

8.8 Solving Radical Equations and Inequalities

◆Skill A Solving radical equations and inequalities without graphs

Recall When you raise both sides of an equation or inequality to a power, you must check for possible extraneous (false) solutions.

◆ Example 1

Solve $\sqrt{8x+1}+1=2x$. Check your solution(s).

◆ Solution

$\sqrt{8x+1}+1=2x$

$\sqrt{8x+1}=2x-1$ *Isolate the radical.*

$(\sqrt{8x+1})^2=(2x-1)^2$ *Square each side.*

$8x+1=4x^2-4x+1$

$4x^2-12x=0$ *Collect like terms and set equal to 0.*

$4x(x-3)=0$ *Factor.*

$4x=0$ or $x-3=0$

$x=0$ or $x=3$

Check. $\sqrt{8\cdot 0+1}+1=2\cdot 0$ $\quad$ $\sqrt{8\cdot 3+1}+1=2\cdot 3$

$2\neq 0$ $\quad$ $6=6$

The only solution is $x=3$.

◆ Example 2

Solve $\sqrt{2-x}\geq 3$. Check with a table.

◆ Solution

$\sqrt{2-x}\geq 3$

$2-x\geq 9$

$-x\geq 7$

Multiply by -1 and reverse the inequality sign.

$x\leq -7$

Check. Let $y=\sqrt{2-x}$ and use a table.

When $x\leq -7$, $y\geq 3$.

X	Y1	
-10	3.4641	
-9	3.3166	
-8	3.1623	
-7	3	
-6	2.8284	
-5	2.6458	
-4	2.4495	

X=-10

Solve each radical equation by using algebra. Check your solution(s).

1. $\sqrt{2x+3}=5$ ____________

2. $x+\sqrt{x-1}=3$ ____________

3. $\sqrt{7-6x}=2-3x$ ____________

Solve each radical inequality by using algebra. Check your solution(s).

4. $\sqrt{x^2-9}\geq 4$ ____________

5. $\sqrt[3]{4x-1}\leq 3$ ____________

6. $\sqrt{3x}+5<3$ ____________

◆Skill B Solving radical equations and inequalities using graphs

Recall If there is more than one radical, you must start by raising both sides to the same power.

◆ Example 1

Solve $\sqrt{2x} + 1 = \sqrt{x - 2}$. Check your solution with a graph.

◆ Solution

$$\sqrt{2x} + 1 = \sqrt{x - 2}$$
$$(\sqrt{2x} + 1)^2 = (\sqrt{x - 2})^2$$
$$2x + 2\sqrt{2x} + 1 = x - 2$$
$$2\sqrt{2x} = -x - 3$$
$$(2\sqrt{2x})^2 = (-x - 3)^2$$
$$4 \cdot 2x = x^2 + 6x + 9$$
$$x^2 - 2x + 9 = 0$$

Use the quadratic formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(9)}}{2(1)} = 1 \pm 2i\sqrt{2}$$

There are no real solutions.

The graphs show that $y_1 = \sqrt{2x} + 1$ and $y_2 = \sqrt{x - 2}$ will never intersect.

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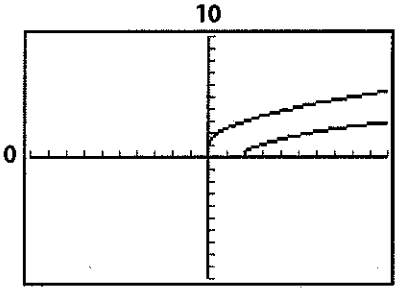
-10  10

-10

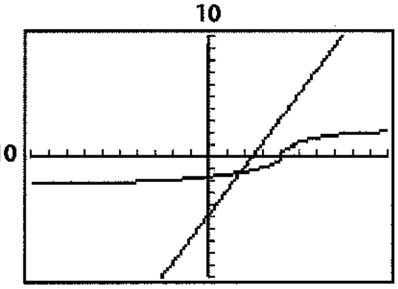
◆ Example 2

Solve $2x - 5 \geq \sqrt[3]{x - 4}$ by graphing.

◆ Solution

Graph $y_1 = 2x - 5$ and $y_2 = \sqrt[3]{x - 4}$. To the nearest tenth, the x-coordinate of the point of intersection is 1.9. When $x \geq 1.9$, the graph of $y_1 = 2x - 5$ is above the graph of $y_2 = \sqrt[3]{x - 4}$. So the solution is approximately $x \geq 1.9$.

10

-10  10

-10

Solve each radical equation by using a graph. Round solutions to the nearest tenth. Check your solution by any method.

7. $\sqrt{x - 3} + 2 = 4$ ____________

8. $\sqrt{x + 4} = 2x - 7$ ____________

9. $\sqrt{3x + 28} = x$ ____________

Solve each radical inequality with a graphics calculator. Round your solution to the nearest tenth, if necessary.

10. $\sqrt{3x + 2} - 1 \geq \sqrt{x + 5}$ ____________

11. $\sqrt{x} \geq \sqrt{2x} - 1$ ____________

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Simplify.

1) $\sqrt[3]{48}$

2) $\sqrt{384}$

3) $\sqrt{147}$

4) $\sqrt[3]{750}$

5) $\sqrt[3]{384n}$

6) $\sqrt{48n^3}$

7) $\sqrt[3]{384b^7}$

8) $\sqrt[3]{-625n^5}$

Solve each equation. Remember to check for extraneous solutions.

9) $\sqrt{4-8n}=6$

10) $\sqrt{22-v}=\sqrt{v-4}$

11) $\sqrt{6-m}=\sqrt{m+10}$

12) $\sqrt{10-2v}=\sqrt{3v-5}$

13) $\sqrt{3x+33}=\sqrt{-17-2x}$

14) $-5=-9+\sqrt{8x}$

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Simplify.

1) $\sqrt{98n^4}$

2) $\sqrt[3]{56m^4}$

3) $\sqrt[3]{80r^6}$

4) $\sqrt{64r}$

Solve each equation. Remember to check for extraneous solutions.

5) $-45 + \sqrt{98n} = -31$

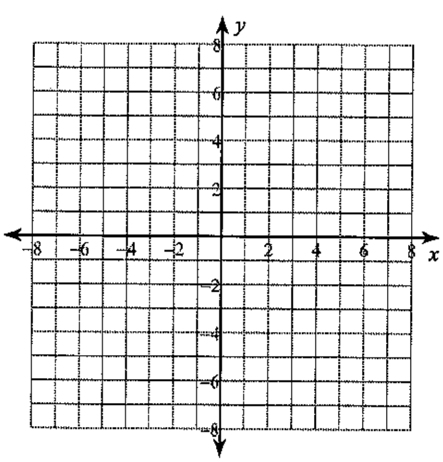
6) $-29\sqrt{17n + 15} = -783$

7) $\sqrt{41 - n} = \sqrt{60 - 2n}$

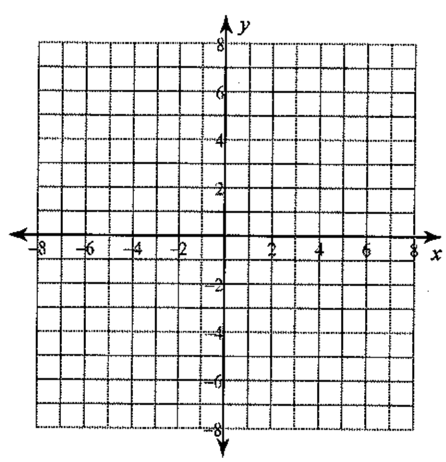
8) $\sqrt{2b + 68} = \sqrt{-25 - b}$

Sketch the graph of each function.

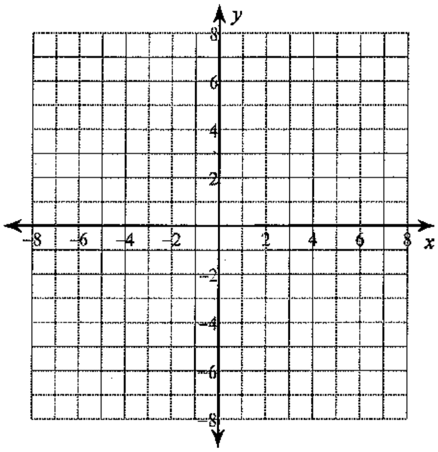
9) $y = \sqrt{x + 3}$



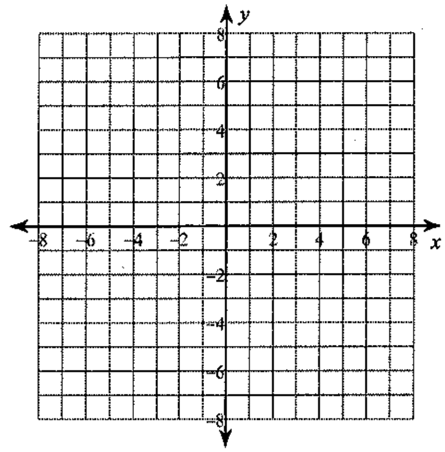
10) $y = \sqrt{x + 1} + 1$



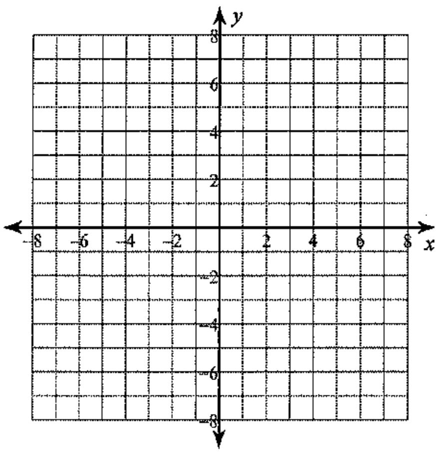
11) $y = \frac{1}{2}\sqrt{x - 1}$



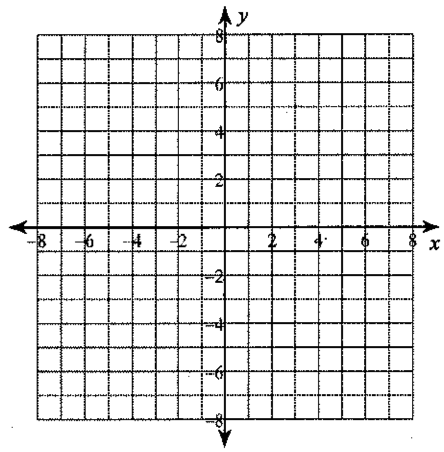
12) $y = \sqrt{x} + 4$



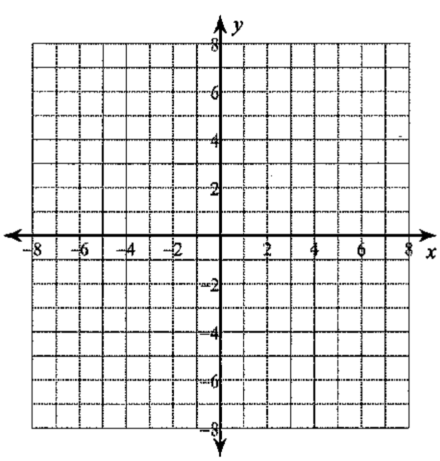
13) $y = -4\sqrt{x - 4}$



14) $y = 3\sqrt{x - 1} - 5$



15) $y = \sqrt{x} + 1$



16) $y = \sqrt{x + 2}$